DYNAMICAL BREAKDOWN AND RESTORATION OF PARITY VERSUS AXIAL ANOMALY IN THREE DIMENSIONS

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Three-dimensional P-, T-invariant gauge nonlinear sigma models with fermions possess different phases where parity may or may not be dynamically broken. Thus, in general there is no axial anomaly (anomalous P-, T-violation) in three dimensions unless background gauge fields with nonvanishing field strengths at infinity are present or when the fermions decouple.

1. Recently much interest was devoted to the investigation of dynamical breakdown of P-(space reflection) and T-(time reversal) invariance in three-dimensional ($D = 3$) gauge theories with massless fermions [1–3]. This $D = 3$ "axial anomaly" was shown to be related to such interesting physical phenomena as fermion number fractionization [2,4] and the quantized Hall effect [4,5]. The content of the "anomaly" may be represented in the following two equivalent forms:

$$\ln \det (\mathrm{i}\not{\nabla}) = \pm \mathrm{i}(16\pi)^{-1}\epsilon^{\mu\nu\lambda}\int \mathrm{d}^3x \{\mathrm{tr}(W_\mu G_{\nu\lambda} - \mathrm{i}\tfrac{2}{3}W_\mu W_\nu W_\lambda) + nA_\mu F_{\nu\lambda}\} + \dots , \tag{1}$$

$$\langle\bar{\psi}\gamma^\mu \tau_A \psi\rangle(x) = \delta \ln \det (\mathrm{i}\not{\nabla})/\mathrm{i}\delta W^A_\mu(x) = \pm n(8\pi)^{-1}\epsilon^{\mu\nu\lambda}G^A_{\nu\lambda}(x) + \dots , \tag{2}$$

and, analogously, for $\langle\bar{\psi}\gamma^\mu\psi\rangle(x) = \delta \ln \det(\mathrm{i}\not{\nabla})/\mathrm{i}\delta A_\mu(x)$, where

$$\not{\nabla} = \gamma^\mu(\partial_\mu + \mathrm{i}A_\mu + \mathrm{i}W_\mu), \quad W_\mu = W^A_\mu \tau_A , \quad G_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + \mathrm{i}[W_\mu, W_\nu], \quad \mathrm{tr}(\tau_A\tau_B) = n\delta_{AB} ,$$

$A, B = 1, \dots, n^2 - 1$ (τ_A are the hermitian SU(n) generators). The P-, T-breaking terms on the RHS of (1) are the well known (topological) gauge invariant mass terms (TGIMT) [6] [‡1] and the dots indicate omitted P-, T-conserving terms.

It was stated in refs. [1,2] that (1), (2) are direct analogues of the standard $D = 4$ ABBJ anomaly of the axial current divergence in the sense that (1), (2) give rise to anomalous violation of P-, T-symmetries in $D = 3$ which cannot be removed through renormalization. The latter statement, however, contradicts in the abelian case the existence of well defined (in the $1/N$ expansion) $D = 3$, P-, T-invariant gauge nonlinear sigma models with (massless) fermions ((GNLSM + F)$_3$) including supersymmetric ones [8–10] which arise as renormalization group fixed points of general $D = 3$ Higgs models with fermions [10].

The aim of the present note is, using (GNLSM + F)$_3$ as a specific example, to clarify the status of P-, T-breakdown in $D = 3$ gauge theories with (massless) fermions. We find:

(i) In $D = 3$ P-, T-symmetries may be dynamically broken in some phases of the theory in the usual manner of spontaneous symmetry breaking – through a nonzero expectation value of an appropriate order parameter ($\langle\langle\bar{\psi}\psi\rangle\rangle$) – and get restored in other phases through (second order) phase transitions (see (6a)–(6d) below). This

‡1 Earlier work can be found in ref. [7].

phenomenon was previously observed in ref. [9] at the instance of dynamical generation of TGIMT in (GNLSM + F)$_3$.

(ii) Eqs. (1), (2) do not contain anomalous terms for any A_μ, W_μ (in the non-abelian case the number of fermion "flavors" should be even, see below), which approach pure gauges at (euclidean space) infinity. (These are precisely the boundary conditions on the euclidean functional integral over A_μ, W_μ). Eq. (2) only holds when A_μ and/or W_μ are external background fields with nonvanishing $F_{\mu\nu}$, $G_{\mu\nu}$ at infinity and when (2) is considered as a zero mass limit of the corresponding massive fermion theory (see (13) below). This is actually the case explicitly considered in refs. [1,2,4,5]. The reason for the noncorrectness of (1), (2) for general A_μ, W_μ is the use of intermediate P-, T-breaking Pauli–Villars regularization in their computation. Instead, we insist on employing P-, T- and BRS symmetry preserving renormalization scheme – a variant of the "soft mass" BPHZL scheme [11] (see (8), (9) below).

(iii) Genuine $D = 3$ axial anomaly arises in the opposite limit of fermion decoupling (i.e. when the physical fermion mass becomes infinitely large), since the limiting effective low energy purely bosonic theory becomes nonrenormalizable (see (14) below).

Of course, the nonperturbative anomaly in $\det(\mathrm{i}\not{\nabla})$ (its change into $(-1)^{|k|}\det(\mathrm{i}\not{\nabla})$ under homotopically nontrivial SU(n) gauge transformations of topological charge k) [1] (remains intact ‡2. However, it is harmless in our case since we may take an even N (number of "flavors").

2. Here we shall consider (GNLSM + F)$_3$ possessing U(N) ("flavor") × U(n) ("color" gauge) internal symmetry ($n < N$) within the $1/N$ expansion [9]. The lagrangian reads:

$$\begin{aligned} \mathcal{L} &= |\nabla_\nu\varphi|^2 + \tfrac{1}{2}\mathrm{i}\bar\psi\overleftrightarrow{\not{\nabla}}^{(\epsilon)}\psi + \lambda_0(4Nn\mu)^{-1}(\bar\psi\psi)^2 + \lambda_1(4Nn\mu)^{-1}(\bar\psi\tau_A\psi)^2 + Nn\mathcal{L}_{A,W}\,, \\ &\varphi^*\varphi - Nn\mu/T = 0\,,\quad \varphi^*\tau_A\varphi = 0\,,\quad \bar\psi\varphi = \varphi^*\psi = \bar\psi\tau_A\varphi = \varphi^*\tau_A\psi = 0\,, \\ \mathcal{L}_{A,W} &= -(4e_0^2\mu)^{-1}F^2_{\kappa\lambda}(A) - (4ne_1^2\mu)^{-1}\,\mathrm{tr}(G^2_{\kappa\lambda}(W)) + \text{gauge fixing term}\,, \end{aligned} \tag{3}$$

with the following notations:

$$(\nabla_\nu\varphi)^k_a = \partial_\nu\varphi^k_a + \mathrm{i}(A_\nu\delta^{kl} + W^{kl}_\nu)\varphi^l_a\,,\quad (\nabla^{(\epsilon)}_\nu\psi)^k_a = \partial_\nu\psi^k_a + \mathrm{i}(\epsilon_0A_\nu\delta^{kl} + \epsilon_1W^{kl}_\nu)\psi^l_a\,,\quad \varphi^*\varphi \equiv \varphi^{*k}_a\varphi^k_a\,,\ \text{etc.}\,.$$

In (3) μ is an arbitrary mass scale, so that all coupling constants T, $\lambda_{0,1}$, $e_{0,1}$ are set dimensionless. $\epsilon_{0,1}$ are the fermion charges in units of $e_{0,1}$. Landau gauge conditions are imposed on A_μ, W_μ. Summation over repeated indices ("flavor" ones $a, b = 1, ..., N$; "color" ones $k, l = 1, ..., n$; adjoint-SU(n) ones $A, B = 1, ..., n^2 - 1$) is understood and the latter will be suppressed. In particular, for $e_{0,1} = \infty$, $\epsilon_{0,1} = 1$, $\lambda_{0,1} = T$ (GNLSM + F)$_3$ coincides with the $D = 3$ supersymmetric GNLSM (in the Wess–Zumino gauge) [8].

(GNLSM + F)$_3$ is invariant under P- and T-reflections:

$$\begin{aligned} &\varphi^{(P,T)}(x) = \eta_{P,T}\varphi(x_{P,T})\,,\quad \psi^{(P,T)}(x) = -\mathrm{i}\eta_{P,T}\gamma_{1,2}\psi(x_{P,T})\,, \\ &(A^{(P)}_\mu)(x) = (A_0, -A_1, A_2)(x_P)\,,\quad (A^{(T)}_\mu)(x) = (A_0, -A_1, -A_2)(x_T)\,, \\ &x_P \equiv (x^0, -x^1, x^2)\,,\quad x_T \equiv (-x^0, x^1, x^2)\,,\quad |\eta_{P,T}| = 1\,. \end{aligned} \tag{4}$$

Note that fermion mass term, as well as the renormalizable polynomial ψ–φ interactions $(\bar\psi\psi)(\varphi^*\varphi)$, $(\bar\psi\varphi)(\varphi^*\psi)$ and their nonabelian counterparts change sign under P-, T-reflections (4).

Construction of the $1/N$ expansion for (GNLSM + F)$_3$ proceeds in the standard manner by first rewriting (3) by means of auxiliary fields (α_0, $\boldsymbol{\alpha}$, σ_0, $\boldsymbol{\sigma}$ are real bosonic and ρ_0, $\boldsymbol{\rho}$ are complex fermionic):

$$\begin{aligned} \mathcal{L}' &= |\nabla_\nu\varphi|^2 - \varphi^*(\alpha_0 + \boldsymbol{\alpha})\varphi + (Nn\mu/T)\alpha_0 - (Nn\mu/\lambda_0)\sigma_0^2 - (N\mu/\lambda_1)\,\mathrm{tr}(\boldsymbol{\sigma}^2) + \tfrac{1}{2}\mathrm{i}\bar\psi\overleftrightarrow{\not{\nabla}}(\epsilon)\psi + \bar\psi(\sigma_0 + \boldsymbol{\sigma})\psi \\ &\quad + \bar\psi(\rho_0 + \boldsymbol{\rho})\varphi + \varphi^*(\bar\rho_0 + \bar{\boldsymbol{\rho}})\psi + Nn\mathcal{L}_{A,W}\,, \end{aligned} \tag{3'}$$

‡2 BRS symmetry preserving renormalization schemes guarantee only invariance under homotopically trivial gauge transformations.

then integrating out $\psi, \varphi_\perp$ (where $\varphi = N^{1/2}\varphi_\| + \varphi_\perp, \varphi^k_{\perp a} = 0, a = 1, ..., n, \varphi^k_{\| a} = 0, a = n+1, ..., N$) in the quantum generating functional:

$$Z[\{j_\phi\}] = \int \mathcal{D}\varphi_\perp \, \mathcal{D}\psi \, \mathcal{D}\varphi_\| \, \mathcal{D}\alpha_0 \ldots \exp\left[i\int d^3x \left(\mathcal{L}' + \sum_{\phi=\varphi,\psi,\ldots,\rho} \phi j_\phi\right)\right] = \int \mathcal{D}\varphi_\| \, \mathcal{D}\alpha_0 \ldots \exp\{iNS_1 + iS_2[\{j_\phi\}]\},$$

$$S_1 \equiv i(1-n/N)\operatorname{Tr}\ln\Delta_B - i\operatorname{Tr}\ln\Delta_F + \int d^3x\,[-\varphi_\|^*\Delta_B\varphi_\| + (n\mu/T)\alpha_0 - (n\mu/\lambda_0)\sigma_0^2 - (\mu/\lambda_1)\operatorname{tr}(\boldsymbol{\sigma}^2) + n\mathcal{L}_{A,W}], \quad (5)$$

$$\Delta_F \equiv i\nabla\!\!\!/^{(\epsilon)} + \sigma_0 + \boldsymbol{\sigma}, \quad \Delta_B \equiv \nabla_\mu\nabla^\mu + \alpha_0 + \boldsymbol{\sigma} + (\bar\rho_0 + \bar{\boldsymbol{\rho}})\Delta_F^{-1}(\rho_0 + \boldsymbol{\rho}),$$

and, finally, expanding S_1 (5) around its constant saddle points: $\hat\varphi_\| \equiv v, \hat\alpha_0 \equiv m_\varphi^2, \hat\sigma_0 \equiv -m_\psi$ (all other fields having zero stationary values). Clearly, m_φ and m_ψ are the corresponding dynamically generated masses of φ and ψ. Note that:

$n^{-1}\langle(\bar\psi\psi)\rangle = (2N\mu/\lambda_0)\langle\sigma_0\rangle = N[-(2\mu/\lambda_0)m_\psi + O(N^{-1})]$.

Solutions of the stationarity equations for S_1 yield the following phase structure (T_c same as in the purely bosonic case [12]):

(a) $T > T_c, 0 < \lambda_0 < T_c$ (P-, T-symmetric "high temperature" phase):

$$V = 0, \quad m_\varphi = 4\pi\mu(1/T_c - 1/T), \quad m_\psi = 0. \quad (6a)$$

(b) $T > T_c$, $\lambda_0 < 0$ or $T_c < \lambda_0 < 2T_c$ ("high temperature" phase with broken P-, T-symmetries, the upper bound on λ_0 comes from stability requirements on the large N effective potential for $(\bar\psi\psi)$):

$$V = 0, \quad m_\varphi = 4\pi\mu(1/T_c - 1/T), \quad |m_\psi| = 4\pi\mu(1/T_c - 1/\lambda_0). \quad (6b)$$

(c) $T < T_c, 0 < \lambda_0 < T_c$ (P-, T-symmetric "low temperature" phase):

$$|V|^2 = \mu(1/T - 1/T_c), \quad m_\varphi = 0, \quad m_\psi = 0. \quad (6c)$$

(d) $T < T_c$, $\lambda_0 < 0$ or $T_c < \lambda_0 < 2T_c$ ("low temperature" phase with broken P-, T-symmetry):

$$|V|^2 = \mu(1/T - 1/T_c), \quad m_\varphi = 0, \quad |m_\psi| = 4\pi\mu(1/T_c - 1/\lambda_0). \quad (6d)$$

The explicit form of the $1/N$ graphical rules and the structure of the (GNLSM + F)$_3$ particle spectrum (containing, in particular, topologically massive W_μ, A_μ-gauge bosons, massive composite fermions ρ_0, $\boldsymbol{\rho}$ and massive composite "color" scalars $\boldsymbol{\sigma}$ in phase (b)) were derived in ref. [9]. Here we shall need only the "free" $1/N$ propagators of A_μ, W_μ:

$$\langle A^\mu A^\nu\rangle^{(0)}(p; e_0, \epsilon_0) = (Nn)^{-1}\, i\,[\mathcal{F}^2(p; e_0, \epsilon_0) - p^2\epsilon_0^4\,\mathcal{C}^2(p)]^{-1}\,[(g^{\mu\nu} - p^\mu p^\nu/p^2)\mathcal{F}(p; e_0, \epsilon_0) + i\epsilon_0^2\,\epsilon^{\mu\nu\lambda}p_\lambda\,\mathcal{C}(p)], \quad (7a)$$

the last term yielding dynamical TGIMT [9],

$$\mathcal{F}(p; e, \epsilon) \equiv -p^2/e^2\mu + |V|^2 + (4m_\varphi^2 - p^2)\tfrac{1}{2}F(p^2; m_\varphi) - m_\varphi/4\pi + \epsilon^2[|m_\psi|/4\pi - (4m_\psi^2 + p^2)\tfrac{1}{2}F(p^2; m_\psi)], \quad (7b)$$

$$\mathcal{C}(p) \equiv 2m_\psi F(p^2; m_\psi) = (4\pi)^{-1}\operatorname{sign}(m_\psi) f(p^2/4m_\psi^2), \quad (7c)$$

$$f(z) = (-z)^{-1/2}\operatorname{arctg}[(-z)^{1/2}], \quad \text{for } z < 0,$$
$$= \tfrac{1}{2}(z^{-1/2})\ln[(1+z^{1/2})(1-z^{1/2})^{-1}], \quad \text{for } z > 0, \quad (7d)$$

$$\langle W_A^\mu W_B^\nu\rangle^{(0)}(p; e_1, \epsilon_1) = \delta_{AB}\langle A^\mu A^\nu\rangle^{(0)}(p; e_1, \epsilon_1). \quad (7e)$$

3. To renormalize the $1/N$ expansion we choose a version of the mass-independent "soft mass" BPHZL scheme

[11] simultaneously suited for all phases (a)–(d) (cf. refs. [8,10]). First, all dimensional parameters entering in propagators and vertices are made "soft":

$$V \to s^{1/2}V, \quad m_{\varphi,\psi} \to m_{\varphi,\psi}(s) = s m_{\varphi,\psi}, \quad \hat{m} \equiv 4\pi\mu(1/\lambda_1 - 1/\lambda_0) \to \hat{m}(s) = s\hat{m},$$

where $\hat{m}$ appears in the $\boldsymbol{\sigma}$-propagator. Secondly, one introduces an additional "soft" mass $z\mu$ only in the φ- and ψ-propagators, serving as an infrared regulator:

$$\langle \varphi_a^k \varphi_b^{*l} \rangle^{(0)}(p) = \delta_{ab}\delta^{kl}[m_\varphi^2(s) + z^2\mu^2 - p^2]^{-1}, \quad \langle \psi_a^k \bar{\psi}_b^l \rangle^{(0)}(p) = \delta_{ab}\delta^{kl}(\not{p} + m_\psi(s))/[m_\psi^2(s) + z^2\mu^2 - p^2]. \tag{8}$$

Here s and z $(0 \leqslant s, z \leqslant 1)$ are auxiliary parameters. Then one employs the standard R-operation recurrence formula (or the "forest formula") [13] with modified subtraction operators $\tau^{\delta(\Gamma)}$:

$$1 - \tau^{\delta(\Gamma)} = (1 - t_z^0 t_{\{p\},s}^{\delta(\Gamma)-1})(1 - t_{z^{-1}}^0 t_{\{p\},s}^{\delta(\Gamma)}), \tag{9}$$

where $t^\delta_{x,\dots,y}$ are the standard Taylor subtraction operators of degree δ around $x = \dots = y = 0$, $\{p\}$ is the set of external momenta and $\delta(\Gamma)$ is the ultraviolet degree of a $1/N$ graph Γ:

$$\delta(\Gamma) = 3 - \tfrac{1}{2}L_\varphi^{\mathrm{ext}}(\Gamma) - L_\psi^{\mathrm{ext}}(\Gamma) - L_{A,W}^{\mathrm{ext}}(\Gamma) - L_\sigma^{\mathrm{ext}}(\Gamma) - 2L_\alpha^{\mathrm{ext}}(\Gamma) - \tfrac{3}{2}L_\rho^{\mathrm{ext}}(\Gamma). \tag{10}$$

$L_\phi^{\mathrm{ext}}(\Gamma)$ denotes number of external ϕ-lines of Γ. After implementing all subtractions one sets $s = 1, z = 0$.

In fact, the "soft mass" scheme was already used in the computation of (7). Note the crucial difference between (8), (9) and the Pauli–Villars procedure. In the P-, T-symmetric phases (a), (c) where $m_\psi = 0$ the additional "soft" mass $z\mu$ appears only in the denominator of $\langle\psi\bar{\psi}\rangle^{(0)}(p)$. Hence, P- and T-invariance are respected by our renormalization procedure. It also respects the "soft" P-, T-breaking in phases (b), (d). This follows directly from the form of the renormalization group equations (RGE) in phases (b), (d):

$$\Big(\mu\partial/\partial\mu + 2\zeta_{\sigma_0} m_\psi \partial/\partial m_\psi + 2[\zeta_{\boldsymbol{\sigma}}\hat{m} + (\zeta_{\sigma_0} - \zeta_{\boldsymbol{\sigma}})|m_\psi|]\partial/\partial\hat{m} + \zeta_\varphi(V\partial/\partial V + V^*\partial/\partial V^*) + \zeta_\alpha m_\varphi \partial/\partial m_\varphi$$

$$- \sum_{r=0,1} e_r^2 \partial/\partial e_r^2 + \sum_\phi \zeta_\phi \int \mathrm{d}^3x\, j_\phi(x)\, \partial/\partial j_\phi(x)\Big) \ln Z[\{j_\phi\}] + \text{contact terms}, \tag{11}$$

where $\zeta_\phi = \zeta_\phi(e_{0,1}; \epsilon_{0,1}) = N^{-1}\xi_\phi^{(1)} + O(N^{-2})$ are the corresponding anomalous field dimensions. In order to obtain the RGE valid for all phases (a)–(d) simultaneously, one should reexpress m_ψ, $\hat{m}$ in terms of the initial coupling constants λ_0, λ_1. These RGE have the same form (11) where the second and the third terms are replaced by:

$$\beta_{\lambda_0}\partial/\partial\lambda_0 + \beta_{\lambda_1}\partial/\partial\lambda_1, \quad \beta_{\lambda_0} = \lambda_0(1 - \lambda_0/T_c)(1 - 2\zeta_{\sigma_0}), \quad \beta_{\lambda_1} = \lambda_1(1 - \lambda_1/T_c)(1 - 2\zeta_{\boldsymbol{\sigma}}). \tag{12}$$

Finally, one can easily verify that the Ward identities of the (spontaneously broken in phases (c), (d)) internal symmetry are preserved by (9) and (8). More details will appear elsewhere.

Thus, we conclude that $(\mathrm{GNLSM} + \mathrm{F})_3$ are well defined within the $1/N$ expansion in all phases (a)–(d). This proves our assertion (i).

4. In the model (3) eq. (2) takes the form according to (8) ($^*G_\mu = \frac{1}{2}\epsilon_{\mu\nu\lambda}G^{\nu\lambda}$)

$$\langle\bar{\psi}\gamma^\mu\tau_A\psi\rangle(x) = (Nn/4\pi)\,\mathrm{sign}(m_\psi)\int\frac{\mathrm{d}^3p}{(2\pi)^3}\exp(-ipx)f(p^2/4m_\psi^2)^*\tilde{G}_A^\mu(p) + \dots$$

(and, analogously, for $\langle\bar{\psi}\gamma^\mu\psi\rangle(x)$) where the dots indicate terms which become P-, T-symmetric in the limit $m_\psi \to 0$. Using elementary properties of the Fourier transform of distributions [14] one can easily check (after rotation to euclidean space) that either

$$Nn(4\pi)^{-1}\,\mathrm{sign}(m_\psi)2m_\psi\int\frac{\mathrm{d}^3p}{(2\pi)^3}\exp(ipx)\,|p|^{-1}\,\mathrm{arctg}(|p|/2m_\psi)^*\tilde{G}_A^\mu(p) \xrightarrow[m_\psi\to 0]{} 0 \quad (\text{for any } {}^*G^\mu(x) = O(|x|^{-\alpha})), \tag{13a}$$

or

$$Nn(4\pi)^{-1}\operatorname{sign}(m_\psi)2m_\psi)2m_\psi\int\frac{d^3p}{(2\pi)^3}\exp(ipx)|p|^{-1}\operatorname{arctg}(|p|/2m_\psi)^*G^\mu_A(p)\xrightarrow[m_\psi\to 0]{}\operatorname{sign}(m_\psi)Nn(4\pi)^{-1}\,{}^*G^\mu_A$$

$$(\text{if } {}^*G^\mu(x)\xrightarrow[|x|\mapsto\infty]{}{}^*G^\mu\neq 0)\,, \tag{13b}$$

where α is an arbitrary positive degree. Thus, assertion (ii) is also proved.

5. Finally, let us consider the opposite limit $|m_\psi|\to\infty$ in phases (b), (d) when $m_\varphi = 0$, $V = 0$ (i.e. $T = T_c$). According to (6) this is equivalent to the limit $\lambda_0\uparrow 0$ which means by (12) infrared scaling limit. The "free" gauge field propagators retain their form (7a), (7e) with:

$$\mathcal{F}(p) = -p^2/e^2\mu + \tfrac{1}{16}(-p^2)^{1/2}\,,\quad \mathcal{C}(p) = \operatorname{sign}(m_\psi)\cdot(4\pi)^{-1}\,,$$

i.e. although the fermions decouple, they leave behind explicit TGIMT with *fixed* constant coefficients (cf. (7a)). In the infrared scaling limit of (3) also $e_{0,1}\to\infty$ (cf. (11)) (recall that in $D = 3$ gauge coupling constants have positive mass dimension). Thus, we get the following effective low energy theory:

$$\mathcal{L}_* = |\nabla_\nu\varphi|^2 + \tfrac{1}{4}\operatorname{sign}(m_\psi)N\epsilon^{\mu\nu\lambda}\{\xi_1\operatorname{tr}[W_\mu G_{\nu\lambda} - i\tfrac{2}{3}W_\mu W_\nu W_\lambda] + n\xi_0 A_\mu F_{\nu\lambda}\} + \text{gauge fixing terms}\,,$$
$$\varphi^*\varphi - Nn\mu/T_c = 0\,,\quad \varphi^*\tau_A\varphi = 0\,,\quad \xi_1 = \xi_0 = 1/4\pi = \text{fixed}\,, \tag{14}$$

whereas by topological reasons ξ_1 must be quantized ($\xi_1 = l/4\pi$, l being integer and $l = 1$ in our case) [6], there is no restriction on ξ_0. In higher orders in $1/N$ the abelian TGIMT will induce according to (10) new ultraviolet divergences which were absent in (3) before the fermion decoupling and which cannot be absorbed through renormalization of ξ_0 since the latter is not a free parameter in (14). ($\xi_0 = 1/4\pi$ is not a fixed point of the corresponding β_{ξ_0}-function, which in the leading $1/N$ order reads [10]: $\beta^{(1)}_{\xi_0} = -N^{-1}8^3\xi_0\{3\pi^2[1+(8\xi_0)^2]\}^{-1}$). Hence (14) is a nonrenormalizable theory.

Thus, we see that true axial anomaly (anomalous violation of P-, T-invariance) in $D = 3$ does actually arise only in the infrared limit in P-, T-nonsymmetric phases of classically P-, T-invariant gauge theories with fermions when the P-, T-breaking is due to a dynamically generated fermion mass.

The infrared limit of $(\text{GNLSM} + \text{F})_3$ (3) in phases (b), (d) (where $m_\psi\neq 0, m_\varphi = 0$) may be viewed as the result of (formal) high-temperature dimensional reduction [15] of $D = 4$ U(N) ("flavor") × U(n) ("color" gauge) Higgs models coupled to *chiral* $D = 4$ fermions. The latter theory is anomalous. Then the nonrenormalizable TGIMT in (14) may be considered as the $D = 3$ counterpart of the $D = 4$ anomaly in the high-temperature limit when the fermions decouple.

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The result for $\beta^{(1)}_{\xi_0}$ (the leading $1/N$ order renormalization group function for the parameter of the abelian topological mass term $(1/4)N\xi_0 \int d^3x\, \epsilon_{\mu\nu\lambda} A^\mu F^{\nu\lambda}$) quoted after eq. (14) is not correct due to the omission of one graph in the previous computation. Adding the contribution of this graph yields $\beta^{(1)}_{\xi_0} = 0$ in accordance with a recently proved general theorem [15] on nonrenormalization of ξ_0 (in gauge theories with massive matter fields). In particular, the model of eq. (14) is renormalizable.

Using the $1/N$ expansion together with the "soft-mass' renormalization scheme described in the letter, which unlike the ordinary perturbation theory is free of infrared divergences in the massless case (for $D = 3$), one can easily extend the proof of ref. [15] to include models containing massless charged matter fields.

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R.U. Khafizov and S.V. Tolokonnikov, Effective charges for the unique first-forbidden β-transitions, Phys. Lett. 153B (1985) 353.

On page 355, the third row of table 1 should read:

$(2^-)^{38}Cl \to {}^{38}Ar(0^+)$	3.86×10^3	5.4×10^2	0.5	5.2×10^3	4.8×10^3 .

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Y. Ne'eman and Dj. Šijački, SL(4,R) world spinors and gravity, Phys. Lett. 157B (1985) 275.

In eqs. (16) and (18), the holonomic version of the anholonomic parameters $e^b{}_a$ and $\alpha^b{}_a$ is $\underline{\{e^b{}_\mu\, (\partial_\rho \xi^\mu)\, h^\rho{}_a\}}$. Eqs. (15) and (17), when multiplied by $(e^b{}_\mu h^\rho{}_a)$ are gauge-equivalent to constant SL(2,C) or $\overline{\text{SL}}(4,\text{R})$ matrices. The expansion based on eq. (19) is unnecessary for the derivation of (20), which results directly from (18).

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